Comput Math Organ Theory (2008) 14: 302–319
DOI 10.1007/s10588-008-9034-2

The dynamics of functioning investigating societal transitions with partial differential equations

Hans de Haan

Published online: 20 September 2008

Abstract In this article a mathematical framework is introduced and explored for the study of processes in societal transitions. A transition is conceptualised as a fundamental shift in the functioning of a societal system. The framework views functioning as a real-valued field defined upon a real variable. The initial status quo prior to a transition is captured in a field called the regime and the alternative that possibly takes over is represented in a field called a niche. Think for example of a transition in an energy supply system, where the regime could be centrally produced, fossil fuel based energy supply and a niche decentralised renewable energy production. The article then proceeds to translate theoretical notions on the interactions and dynamics of regimes and niches from transition literature into the language of this framework. This is subsequently elaborated in some simple models and studied analytically or by means of computer simulation.

Keywords Societal transitions · Partial differential equations · Strategic niche management

1 Introduction

This article proposes a mathematical framework for studying societal transitions. Societal transitions are viewed as fundamental change in the functioning of a societal system. Functioning is the way a societal system meets societal needs. For example a transition in an energy supply system could be changing from centralised production of energy using fossil resources to a decentralised system where consumers

J. de Haan (✉)
Dutch Research Institute for Transitions, Faculty of Social Sciences, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR, Rotterdam, The Netherlands
e-mail: j.dehaan@fsw.eur.nl
url: http://www.eur.nl/fsw/staff/homepages/dehaan/

can also deliver energy to the grid, all based on renewable resources. These large-scale changes take place on several dimensions, like type of resources, technology involved, legislation, attitude towards the form of production and the degree of centralisation. It is therefore not difficult to imagine that a transition can easily span several generations.

In transition studies it is not uncommon to describe this in terms of a regime change. The initial regime in this case corresponds to the energy system functioning on the basis of central production, fossil fuels, etc. The new regime is then thought of as growing out of a niche, a nucleus of innovative or otherwise different functioning. In this example a constellation around renewable energy sources can be considered a niche. How and under what conditions such a niche scales up and replaces the incumbent regime is therefore an important matter in a societal transition.

Since the regime by definition dominates the functioning of the societal system, and as such controls the infrastructures and determines the current discourse, it is a constellation with power. Compared to the regime, niches function on smaller scales, presumably with novel approaches and experimental technology. The lack of power of a niche can be partially compensated by societal support. For instance, financial support might be injected in the form of subsidies, political support could be given in the form of benign legislation or there might simply be a small market for the niche already.

This article mathematically frames societal transitions in the terminology introduced above. The aim of this article is to make it possible to investigate—in a mathematical way—questions like: How does support influence the scaling up of niches? Or: Does the functioning or regimes and niches change if there is competition on one of the several dimensions? Is a niche more likely to scale up if it functions in a way similar to the regime, or just the opposite? The reason to investigate those questions mathematically is that it allows one to rigorously track the consequences of certain hypotheses. Although the elaborations might be complicated, the assumptions can be directly tested for their consequences. Another advantage of a mathematical approach is that any problem, once cast in equations becomes comparable to other problems that share the same mathematical form, although the original questions had nothing in common.

So now it becomes necessary to represent regimes and niches in some mathematical form. Apparently regimes and niches are constellations with properties in possibly various dimensions. Let such dimensions be parameterised by some real parameter φ, and for ease of exposition let the number of dimensions be limited to one. For example a regime or niche can be thought of as taking a position on the φ-axis running from fossil fuel only to renewable or bio fuel only. Again taking the initial example the regime would be distributed on the 'fossil side' of the φ-axis and a bio fuel niche on the other, 'renewable' side. This is of course a caricature, but in several dimensions such a characterisation is not unthinkable.

This has similarities with the spatial approaches in political science, e.g. (Downs 1957; Farlow 1982; Kollman et al. 1992; Laver 2005). In those approaches a political party has coordinates along several political axes, like left-right, conservative-liberal, etc. In this approach for societal transitions one can take this a step further. Instead of assigning a regime or niche to a point on the φ-axis, the constellation becomes a distribution over that axis. In other words, regimes and niches become functions of φ, or in yet other words, they become fields defined on a functioning space. This has the extra advantage of allowing the surface area under these functions to represent the power of the regime or niche. To illustrate this again with the energy example, the regime would be a large blob on the fossil side and a bio fuel niche a small blob on the renewable side.

Somewhat more on this interpretation of power maybe. If the surface area under a regime or niche function represents its power, then the value of its function at a certain point also has an interpretation, namely it is the specific power of the constellation over a specific form of functioning. One can easily imagine that this interpretation becomes relevant when a niche is in competition with a regime. Still more on power. Apparently the sum total of the surface areas under the regime and niches is the total power present in the societal system. If one considers the representation of the regimes and niches as complete, that is all the necessary dimensions are taken into account, then one can defend that the total power in the societal system is constant and merely redistributed over the several constellations. This implies a conservation law which can be quite instrumental[1] and in fact the mathematical framework in this article depends on it.

Obviously such a conservation law is also a conceptual limitation. It is a so-called zero-sum interpretation of power, described and criticised by Parsons already. In this article power is the integral over the functioning and functioning is the way a societal need is met. If conservation of power is assumed this implies that all possible such ways are already present and no new ways enter the system, the possibilities for transitional change therefore are limited to a the shifting of power over the φ-axis, for instance by growth of a niche or movement of the regime. A limitation of similar nature will arise when the general form of the evolution equations (8) is presented. This limits the systems that can be modelled in this framework to investigating the scaling up of functioning that is already known, e.g. known societal practice or available technology. Moreover, the models presented in this article are especially concerned with the initial stages of societal transitions.

The article will now continue to expose the formalism and how hypotheses on transition dynamics are cast in this formalism. Clearly, when all the concepts are cast in the form of distributions evolving and interacting, the mathematics will be that of coupled partial differential equations (pde's). For clarity and mathematical tractability the number of dimensions in this article is limited to one, although there is in principle no restriction there. The merits and challenges of the framework will be discussed at length at the end of this article, suffice it to say that methodologically the approach is akin to that of pattern formation in physics,[2] chemistry, biology and ecology, but also to the classical field theories of physics (Landau and Lifshitz 1951), which opens a wide body of knowledge for the emerging field of transition science. The remainder of the article will elaborate some simple consequences of the framework and illustrate its usefulness in several simple model settings.

[1] For this and other debates on power see e.g. (Haugaard 2002). For a more complete review of power in the context of transition studies see (Avelino 2007).

[2] The field is vast and ever growing, some of the authors favourites are (Cross and Hohenberg 1993; van Saarloos 2003).

It needs to be borne in mind that the mathematical framework presented is principally meant to rigorously perform thought experiments on the initial stages of societal transitions. The motivation for the form of the equations and interpretations of their parameters will therefore mostly be cast in a theoretical form. Notwithstanding that links will be attempted to make to possible operational forms of the model to real-life cases by means of examples. Furthermore the models presented and used in the elaborations are not as much linked to, or drawn from, actual societal transition cases but rather the simplest non-trivial examples of how the mathematical framework could be employed.

2 Formalism

The introduction already suggested that the form the regime and niches will take would be that of functions of φ. To help intuition somewhat here, the φ-axis would represent the various ways a certain societal need can be met. For instance if the societal need is personal (auto)mobility, φ could range from fossil fuel based solutions to climate neutral solutions. Along the axis one would then find on the one end cars and infrastructure around internal combustion engines, next to ict-guided highways, carpooling solutions, and bio-fuel based solutions moving all the way to, for instance, electric cars powered with green electricity on the other side of the axis. Close to each other on the axis would always be societal practices or technological solutions that are akin or in another sense "close". For an operational form the continuous axis could be sampled to represent the available ways the societal need can be met, after which a regime could be identified as well as a niche representing some promising innovative practice.

The interpretation of surface area as power is easier if one demands that the functions be positive, which also avoids difficult interpretation of what would be negative functioning. Therefore, the regime, niches and possible other constellations are represented by fields defined upon the specific functioning φ and time t with values in $\mathbb{R}^+$. Where φ is a real parameter on some domain $\Phi \subset \mathbb{R}$ and $t \in [0, \infty)$. This invites the following notation for the regime and niches:

The regime:

$$R(\varphi, t) \to \mathbb{R}^+. \tag{1}$$

A niche:

$$n(\varphi, t) \to \mathbb{R}^+. \tag{2}$$

It will sometimes prove to be convenient to speak of an arbitrary or general constellation (regime, niche) i, which will then be denoted as $c_i(\varphi, t)$. The functioning of the societal system as a whole was already defined as the sum of the functioning of all its constellations, which then is obviously $\sum_i c_i(\varphi, t)$.

The fraction of the functioning of the societal system that a certain constellation contributes to the whole is an interpretation of its power. This leads to the following definition of the power π_i of a constellation i:

$$\pi_i = \int_\Phi \mathrm{d}\varphi c_i(\varphi, t). \tag{3}$$

Now that power is defined in terms of functioning it is possible to postulate the conservation of power as follows:[3]

$$\partial_t \int_\Phi d\varphi \sum_i c_i(\varphi, t) = \partial_t \sum_i \pi_i = \partial_t \Pi = 0 \tag{4}$$

implicitly defining Π as the total power.

It is obvious that the more two constellations are alike in functioning, the more their functioning will overlap. The power that is represented in this overlap is therefore a convenient measure of the likeness or closeness of niches and niches and the regime. The power overlap of two constellations c_i and c_j is

$$\pi_{ij} = \int_\Phi d\varphi c_i \theta(c_j - c_i) + c_j \theta(c_i - c_j), \tag{5}$$

where $\theta(x)$ is the Heaviside function, unity for positive arguments and zero for negative arguments.

After these preliminary matters of definition and notation it is now possible to discuss how regimes and niches change in time. In other words, what is the form of the evolution equations for the constellations? One realises that a regime or niche, when left 'alone' will still have a time evolution caused by its internal dynamics. Constellations are after all societal constructs comprised of actors, institutions and such. Apart from the internal dynamics regimes and niches are *interacting*. Interaction can be the result of competition, political influencing, using the same resources or infrastructures and much more. Interaction of course results in the coupling of the evolution equations. In any case, these two influences on the dynamics of constellations will be separated in the equations. So a general form for an arbitrary constellation i takes the form of:

$$\partial_t c_i(\varphi, t) = \mathcal{F}_i[c_i] + \mathcal{I}_i[c_i, c_j], \tag{6}$$

where $\mathcal{F}_i$ is a differential operator representing the internal dynamics of constellation c_i itself, $\mathcal{I}_i$ with $j \neq i$ is the interaction term that in general will depend on some or all of the other constellations.

The conservation of power leads to the idea that what one constellation gains in power must be lost by the others. This demand can be a direct consequence of the forms of $\mathcal{F}_i$ and $\mathcal{I}_i$. If this is not the case balancing terms $\mathcal{B}_i$ need to be introduced that in turn will depend on $\mathcal{F}_i$ and $\mathcal{I}_i$, like in

$$\partial_t c_i(\varphi, t) = \mathcal{F}_i[c_i] + \mathcal{I}_i[c_i, c_j] - \mathcal{B}_i[c_i, \mathcal{F}_j, \mathcal{I}_j]. \tag{7}$$

If no direct interactions are present the $\mathcal{B}_i$ will then still provide an indirect coupling.

For example, one niche and one regime with interactions that need balancing give rise to a set of coupled pde's of the following form:

$$\begin{cases} \partial_t R(\varphi, t) = \mathcal{F}_R[R] + \mathcal{I}_R[R, n] - \mathcal{B}_R[R, \mathcal{F}_n, \mathcal{I}_n] \\ \partial_t n(\varphi, t) = \mathcal{F}_n[n] + \mathcal{I}_n[n, R] - \mathcal{B}_n[n, \mathcal{F}_R, \mathcal{I}_R]. \end{cases} \tag{8}$$

[3]For ease of notation the partial derivative with respect to time is denoted as ∂_t.

One serious limitation of this approach which is immediately clear from (8) is that a constellation is stuck to its $\mathcal{F}_i$. This means that niches can scale up but never become regimes within the frame of the equations and neither can a regime be dethroned in this sense. This a priori limits the approach to the initial stages of transitions or the modelling needs to be done in several phases.

2.1 Internal constellation dynamics

In times where little societal change is occurring the functioning of the societal system still evolves. Thus, each constellation, regime or niche, when left on its own will still evolve over time. Describing this evolution entails translating some theoretical knowledge and assumptions in mathematical terms.

One realisation is that all constellations, be they regime or niches, are expected to smoothen out the different aspects of their functioning. It is, for example, intuitively clear that a certain niche will next to its 'core-business' (say hydrogen energy storage) have an interest in related functioning (say solar energy capture). Moreover a constellation will try to integrate the various aspects of its functioning in a smooth, self-consistent whole. Much literature on regime and niche development elaborate on this development of such constellations to e.g. a self-consistent set of rules or dominant designs like (Nelson and Winter 1977; Kemp et al. 1998). That structuration and striving for self-consistency is also a more general realisation for soci(et)al systems is discussed at length in (Giddens 1984; Luhmann 1984), where in the abstract is discussed how action and structures are recursively shaping each other and how social systems develop a tendency to reproduce themselves.

This kind of dynamics appears to correspond to a diffusive behaviour of constellations, that is the $\mathcal{F}_i$ are expected to have a form like

$$\mathcal{F}_i = \beta_i \partial_\varphi^2 c_i. \tag{9}$$

It is no coincidence that innovation literature also speaks of 'diffusion of innovation' (Rogers 2003), which is readily generalised from technological to societal innovation. Another realisation is that innovation is likely to be found at the 'edges' of the constellations. In other words if constellations attempt to make their functioning self-consistent and smooth, innovation is to expected on the edges. This is often argued as a reason for the incremental character of innovation (Levinthal 1998) and the necessity for (societal) innovation to emerge in niches[4] (Rotmans 2005; Kemp et al. 1998). This also is a mathematical characteristic of diffusive behaviour if the initial form of the constellation is sufficiently localised as any text on pde's can testify e.g. (Farlow 1982).

Although both the regime and niches apparently exhibit diffusive behaviour in the evolution of their functioning they have quite different characters. One expects the regime to be a stable constellation. The regime for various reasons already alluded to above—vested interests, technological lock-in, etc.—is expected to stick to, and

[4]For a review of the niche concept in technological change literature see (Schot and Geels 2007).

optimise its current functioning. This is also recognised in other uses of the regime concept, like policy regimes (Wilson 2000) and urban regime theory (Mossberger and Stoker 2001). From niches, conversely, a more dynamical behaviour is expected. Since they are the loci of novel or deviant functioning, and therefore have larger tendency for experimentation, their functioning is expected to be more mobile and prone to spread out. This spreading out of the functioning of niches is referred to as 'broadening' by van den Bosch and Taanman (2006).

To distinguish these characteristics mathematically the diffusion constant becomes the relevant parameter. The diffusion constant, in this context referred to as the 'broadening parameter', is assumed to be small for a regime and relatively large for niches, in any case

$$\beta_R < \beta_n. \tag{10}$$

Different values of the parameter β can also be interpreted as the strategies of a constellation, similar to the strategies in spatial elections of Laver (2005). A small β would correspond to an 'aggregator' strategy and a large β to a 'hunter' strategy.

To assess the broadening parameter of an actual regime or niche in a case study one needs to first have an operational form for the φ-axis, like suggested in the beginning of this section with the personal mobility example. After that one can investigate, either by extrapolating historical data or by making some ballpark estimation, how the regime or niche will broaden their functioning by offering more ways to meet societal needs.

2.2 Constellation interactions

The interaction terms are a different matter altogether, several views are possible of how constellations interact and how regimes are taken over in the course of transitions. Most theorising however does not explicitly speak of interaction between niches and regimes, except perhaps pillar theory (de Haan 2007) and discuss the intrinsic growth of niches and regimes, again see e.g. (Geels 2002; Kemp et al. 1998). The transition in such views takes place when the niche has become the new most powerful constellation, for instance in terms of market share.

In the following the distinction will be made between *direct* and *indirect* interaction. Direct interaction will have an explicit interaction term $\mathcal{I}_i$, featuring constellations other than i, whereas with indirect interaction everything is mediated by the balancing terms $\mathcal{B}_i$. One of the consequences of this is that the local properties of a regime or niche, that is the form of their functions, is hardly relevant in the indirect interaction cases.

2.2.1 Indirect interaction

A mathematical representation of indirect interaction is possible through the demand of conservation of total power. The regime has an intrinsic tendency to grow but stay close to its current forms of functioning. The latter was reflected in its small value of β_R the former translates to a growth term proportional to itself. That is, added to $\mathcal{F}_R$ is a term proportional to R resulting in:

$$\mathcal{F}_R = \beta_R \partial_\varphi^2 R + \sigma_R R, \tag{11}$$

where σ_R is some positive constant.

The growth of a constellation, and therefore the interpretation of the parameters σ can be measured in various ways. One could think for example of market shares, percentage of users adopting one or the other societal practice

The greater tendency for exploration for a niche was already captured by a larger value of β_n. A desire to grow can however not be captured by a simple self-proportional growth term like in the regime case. Especially strategic niche management but also pillar theory point out that niches are vulnerable and often need societal support. This support can take various forms, be they political in the form of helpful legislation or financial in the form of subsidies. For this it is convenient to introduce the support function, which is akin to the support canvas in Bergman et al. (2008).

$$S(\varphi), \tag{12}$$

depending on φ to reflect that support for specific forms of functioning is possible. For a niche $\mathcal{F}_n$ will according to the above have the following form:

$$\mathcal{F}_n = \beta_n \partial_\varphi^2 n + \sigma_n n S. \tag{13}$$

The term σ_n is the intrinsic potential for a niche to grow and the less a niche needs support the larger σ_n is expected to be. The growing of a niche will be called 'scaling up' following van den Bosch and Taanman (2006) and the σ_i's will be denoted as 'upscaling parameters'.

An example societal system with one regime and a niche now takes the form of the following set of equations:

$$\begin{cases} \partial_t R(\varphi, t) = \beta_R \partial_\varphi^2 R + \sigma_R R - \mathcal{B}_R \\ \partial_t n(\varphi, t) = \beta_n \partial_\varphi^2 + \sigma_n n S - \mathcal{B}_n. \end{cases} \tag{14}$$

The balancing terms $\mathcal{B}_i$ necessary for the conservation of power and thus the implicit interaction can still take several forms. The demand $\partial_t \Pi = 0$ is equivalent to

$$\int_\Phi \mathrm{d}\varphi \left(\sum_i \mathcal{F}_i - \mathcal{B}_i \right) = 0 \tag{15}$$

because of (4) and with the $\mathcal{I}_i c_i$ equal to zero. This leads to

$$\sum_i \left(\int_\Phi \mathrm{d}\varphi \mathcal{F}_i - \int_\Phi \mathrm{d}\varphi \mathcal{B}_i \right) = 0 \tag{16}$$

and

$$\sum_i \int_\Phi \mathrm{d}\varphi \mathcal{B}_i = \sum_i \int_\Phi \mathrm{d}\varphi \mathcal{F}_i. \tag{17}$$

For the simple system of equation (14) the following $\mathcal{B}_i$ keep the total power conserved:

$$\mathcal{B}_1 = \mathcal{F}_2, \tag{18}$$

$$\mathcal{B}_1 = \frac{c_1}{\int_\Phi \mathrm{d}\varphi c_1} \int_\Phi \mathrm{d}\varphi \mathcal{F}_2. \tag{19}$$

Other terms are possible as well, obviously. The first form of the balancing term simply takes the power from the one constellation at the specific functioning where the other gains it, the second form collects the gain of the one and takes it from the other in proportion to its functioning. A disadvantage is that these $\mathcal{B}_i$ do not necessarily keep the constellations positive.

2.2.2 *Direct interaction*

Perhaps a more realistic view on societal dynamics is that the constellations interact directly. This view entails a sort of 'competition for power over specific functioning'. When a constellation expands its functioning in this picture there will be competition in the regions where it overlaps with another. The difference between niche and regime, in that the former depends partially on support and the latter can grow independently, is retained.

This results in interaction terms that are reminiscent of those of spatial predator-prey, bacterial growth and even some vortex-fluids interaction models (Baggio et al. 2004). For instance, for the regime an $\mathcal{I}_R$ is proposed of the form:

$$\mathcal{I}_R R = \sigma_R R n. \tag{20}$$

And for niches of the form:

$$\mathcal{I}_n n = \sigma_n n R(S - R), \tag{21}$$

where the $S - R$ term reflects the notion raised in pillar theory (de Haan 2007) that niches emerge where the power exercised by the regime is low and support is present.

Since the interaction is direct and local, the obvious choice for the $\mathcal{B}_i$ is the form (18). An example system of a regime and one niche now becomes:

$$\begin{cases} \partial_t R(\varphi, t) = \beta_R \partial_\varphi^2 R + \sigma_R R n - \sigma_n n R(S - R) \\ \partial_t n(\varphi, t) = \beta_n \partial_\varphi^2 n + \sigma_n n R(S - R) - \sigma_R R n. \end{cases} \tag{22}$$

It is also possible to subsume the $\mathcal{B}_i$ in the interaction terms, if the support is interpreted as giving a niche—locally—more power, as is the case for instance with subsidies and legislative measures. An expanding niche then competes with the regime with an effective specific power nS. The straightforward implementation of this would be that the niche gains if $nS > R$ and the regime if $nS < R$. The interaction term would look like $\gamma n R(nS - R)$. A simple example system in this case becomes:

$$\begin{cases} \partial_t R(\varphi, t) = \beta_R \partial_\varphi^2 R - \gamma n R(nS - R) \\ \partial_t n(\varphi, t) = \beta_n \partial_\varphi^2 n + \gamma n R(nS - R), \end{cases} \tag{23}$$

where γ is a 'phenomenological' interaction parameter. It is more difficult to speak of upscaling parameters here, although γS can be interpreted as an effective upscaling parameter. In fact, it could be interpreted as a parameter measuring the effectiveness of the competition over certain forms of functioning. Since it has the dimension of time divided by functioning squared, the interpretation could be that of the speed with which the constellations compete over functioning. As an example one could think of hybrid cars being competed over by both the regime constellation around fossil fuel as by the green mobility niche that was previously into electric cars only.

3 Elaborations

3.1 Simple consequences of conservation of power

The conservation of power in itself already has its consequences for the dynamics. Take for instance a very simple system consisting of two constellations, a regime and a niche with some support. The dynamical system becomes

$$\begin{aligned} \partial_t R(\varphi,t) &= \beta_R \partial_\varphi^2 R + \sigma_R R - \frac{R}{\int_\Phi \mathrm{d}\varphi R} \int_\Phi \mathrm{d}\varphi (\beta_n \partial_\varphi^2 n + \sigma_n n S), \\ \partial_t n(\varphi,t) &= \beta_n \partial_\varphi^2 n + \sigma_n n S - \frac{n}{\int_\Phi \mathrm{d}\varphi n} \int_\Phi \mathrm{d}\varphi (\beta_R \partial_\varphi^2 R + \sigma_R R). \end{aligned} \tag{24}$$

To explore the consequences of power conservation the explicit dependency on the specific functioning becomes irrelevant and if one integrates out the dependence on φ an expression in terms of the power of the constellation is obtained:

$$\begin{aligned} \partial_t \pi_R(t) &= \sigma_R \pi_R - \sigma_n \bar{s} \pi_n, \\ \partial_t \pi_n(t) &= \sigma_n \bar{s} \pi_n - \sigma_R \pi_R. \end{aligned} \tag{25}$$

It is assumed that the diffusive term integrates to zero because of the boundary conditions and furthermore the contribution of the support function is captured in the effective support $\bar{s}$, which is equivalent to assuming $S(\varphi)$ independent of φ.

Equations (25) are a simple linear system that can be solved exactly. In matrix notation they become

$$\partial_t \boldsymbol{\pi} = \begin{pmatrix} \sigma_R & -\sigma_n \bar{s} \\ -\sigma_R & \sigma_n \bar{s} \end{pmatrix} \boldsymbol{\pi}, \tag{26}$$

with

$$\boldsymbol{\pi} \equiv \begin{pmatrix} \pi_R(t) \\ \pi_n(t) \end{pmatrix}. \tag{27}$$

The general solution is

$$\boldsymbol{\pi}(t) = \mathbf{v}_1 k_1 e^{\lambda_1 t} + \mathbf{v}_2 k_2 e^{\lambda_2 t}, \tag{28}$$

where the v_i are the eigenvectors corresponding to the eigenvalues λ_i. The k_i are constants to be determined through the initial conditions. After some linear algebra the following is obtained for the time dependence of the power of the regime and the niche:

$$\begin{aligned} \pi_R(t) &= \tfrac{\sigma_n \bar{s}}{\sigma_R} k_1 - k_2 e^{(\sigma_R + \sigma_n \bar{s})t} \\ \pi_n(t) &= k_1 + k_2 e^{(\sigma_R + \sigma_n \bar{s})t}. \end{aligned} \tag{29}$$

To see the implications it is necessary to supply some initial conditions. In a typical system with an incumbent regime and an upcoming niche the power of the regime is large in comparison to that of the niche. It is then interesting to investigate how the initial power of the niche influences the evolution. To model this one assigns a value to the regime at $t = 0$, which can without loss of generality be set to unity, and leaves the initial power of the niche as a variable, like

$$\begin{aligned} \pi_R(0) &= 1, \\ \pi_n(0) &= \epsilon. \end{aligned} \tag{30}$$

Typically one assumes $\epsilon \ll 1$. For the k_i one now finds

$$k_1 = \frac{1+\epsilon}{1+\left(\frac{\sigma_n \bar{s}}{\sigma_R}\right)} \quad \text{and} \quad k_2 = \frac{-1+\left(\frac{\sigma_n \bar{s}}{\sigma_R}\right)\epsilon}{1+\left(\frac{\sigma_n \bar{s}}{\sigma_R}\right)}. \tag{31}$$

A quick glance at (29) learns that for the niche to successfully scale up to a level where its power will exceed that of the regime, k_2 needs to be positive. Since all upscaling parameters are positive this implies the *upscaling condition*

$$(\sigma_n \bar{s})\epsilon > \sigma_R. \tag{32}$$

The interpretation of this condition is readily found. No matter how powerful the regime is in the beginning—that is, no matter how small ϵ—if its *upscaling potential* is large enough it will become more powerful than the regime at a certain point in time. Of course, like alluded to earlier, before this point is reached the modelling assumptions become inconsistent since the niche is assumed to adopt regime-like behaviour when it becomes a serious competitor for the incumbent regime.

3.2 The effect of functioning on transition paths

From the previous it is clear that in the cases with indirect interactions the functioning might influence the course of a transition, but not the eventual outcome. This is because it is possible to integrate out the dependence on φ. In these cases the end state depends on the initial conditions in a simple way, namely via an upscaling condition. Therefore, to appreciate how the functioning of constellations impacts the course of a transition it is insightful to employ direct interactions.

In the following an attempt is made to explore how the initial form and distribution of constellations lead to different transitions paths and outcomes. For this a simple system of one niche in interaction with a regime is used with a support function that is independent of φ. The influence of support on transition paths will be explored later on. The example system (23) will be studied here:

$$\begin{cases} \partial_t R(\varphi, t) = \beta_R \partial_\varphi^2 R - \gamma n R(nS - R) \\ \partial_t n(\varphi, t) = \beta_n \partial_\varphi^2 n + \gamma n R(nS - R), \end{cases} \tag{33}$$

with S taken independent of φ.

For the initial distributions of the regime and the niche Gaussian functions will be used, broad ones for the regime, more sharply peaked ones for the niche. To reflect the typical power distribution before a transition the regime will initially have approximately four to five times as much power as the niche.[5]

In systems with direct interactions, competition produces relevant dynamics. Therefore it is important to know how much power 'overlaps' initially, so to speak. The measure for this was already given by (5), which gives the power overlap π_{ij} of two constellations c_i and c_j.

The behaviour of (23) was studied with Mathematica 5.1. The initial position of the regime was the same throughout and the initial position of the niche varied. This was repeated for various values for the support function. The boundary conditions were Neumann's, that is the derivatives were held zero. A spatial domain running from -100 to 100 was used. As said, the regime's initial condition was constant and of the following form:

$$R(\varphi, 0) = 5e^{-\frac{(\varphi-25)^2}{200}}. \tag{34}$$

Resulting in a power π_R of 125.3. The form of the niche function was

$$n(\varphi, 0) = 5e^{-\frac{(\varphi-\varphi_0)^2}{10}}, \tag{35}$$

that is, less wide and less powerful with $\pi_n = 28.0$, see Fig. 1. To reflect the theoretical notion that the regime is more rigid than a niche a factor of five was ini-

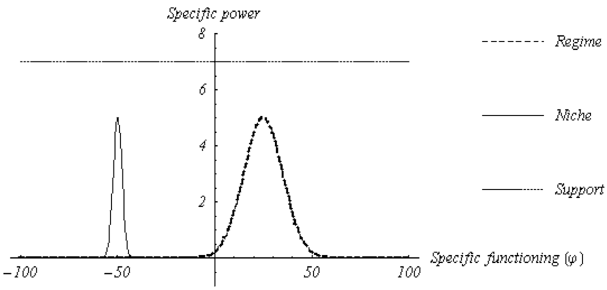

Fig. 1 Typical initial conditions

[5] Of course this is disputable, since there is no agreed quantitative measure of power in general. However, within the framework sketched in the beginning of this article such a measure exists and then it is somewhat reasonable to say a factor of four to five is a significant difference.

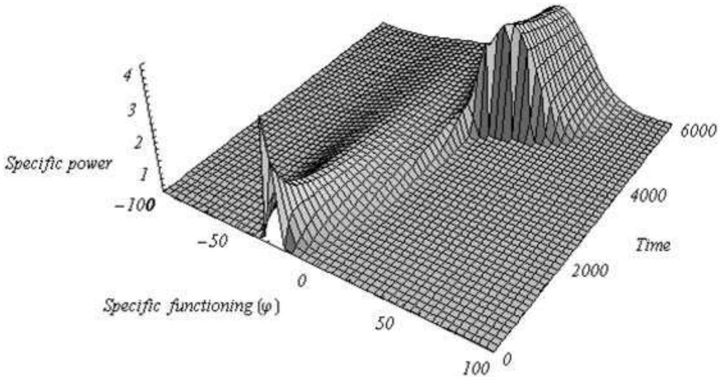

Fig. 2 Evolution of a niche, gaining power by changing its functioning

tially chosen between their broadening parameters, specifically $\beta_R = 1 \times 10^{-2}$ and $\beta_n = 5 \times 10^{-2}$.

As far as the influence of the support function is concerned the results were straightforward, more support increases the possibility for the niche to grow at the expense of the regime. More specifically it was found that between $S = 4$ and $S = 3$ the chances of a niche turn. If it has less support than $S = 3$, the niche has no chance. Given enough support, however, it proved to improve the chances of a niche to rise to power, when it starts *close* to, e.g. has more overlap power with, the regime.

This raises the following question: What exactly is to be considered a successful transition in this representation of a societal system? Is it a niche becoming the new most powerful constellation or is it a significant shift in the entire functioning of the societal system? Both views are present in the literature. The 'niche becoming the new regime' picture more prevalent in the socio-technical transition literature and the 'significant shift' picture more natural for transitions to sustainability. This is important to keep in mind as the functioning of the niche in becoming the most powerful constellation changed profoundly. To state this more exact, the median of the niche constellation in the process of the transition shifted to almost the median of the regime, like for instance in Fig. 2.

3.3 Behaviour and strategies of constellations

Or maybe this is because of the flexible behaviour of niche constellations? Maybe the larger value of the broadening parameter (it is a diffusion parameter, after all) is to blame. In any case it is interesting to now how the broadening parameter values influence the course of a transition. The value of β can also be interpreted as a strategy of the constellation, where low values correspond to rigid, but also robust, behaviour and higher values to a flexible behaviour that also makes it more vulnerable. The influence of the broadening parameter was therefore studied as well, keeping in mind the effects on the total functioning of the societal system.

To test this the system was prepared in a similar way as before with a niche functioning in a sufficiently different region than the regime (overlap of order 10^{-8}, zero) and the broadening parameter was varied from 0.01, which is the regime value, to 0.1.

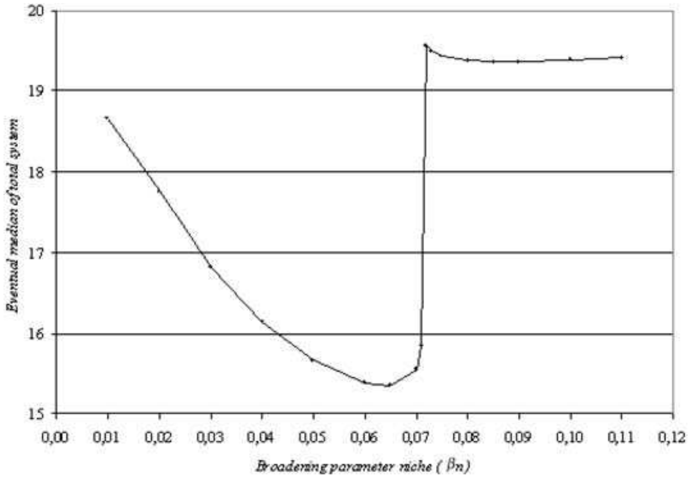

Fig. 3 Median of total functioning as function of β_n

This again for different values of the support, in principle enough for a niche to become the most powerful constellation. In general it is observed that a high value of β_n makes the niche flexible but vulnerable. When β_n is decreased towards more rigidity nothing much happens for the end state until a certain, apparently critical, value is reached after which the niche takes over the dominant position in the societal system.

The peculiarity lies in decreasing the value of β_n even further. An extremely rigid—regime like—niche, does not change its functioning much until its slowly spreading out tail meets that of the regime. If this happens it can grow rapidly if given enough support. The effect however is that it grows in the region of the incumbent regime. An interpretation could be the following, if a niche is very loyal to its principles, or sticks to it's core business it remains the same for a long time but when the opportunity comes forth it is seized, with change of 'character'. This of course has consequences for how the total functioning changes as result of such a transition. Figure 3 shows how the eventual median functioning of the entire system varies as a function of the broadening parameter of the niche.

Note that even at its most extreme the median of the total functioning does not change dramatically. Given that the initial median functioning of the entire system was 22.2, and the niche had its median at −50, a new median at approximately 15 does not appeal to the idea of fundamental change. Nevertheless, an optimum broadening parameter appears to exist which is larger for larger values of the support. This implies that more support allows a niche to be more flexible, which intuitively makes sense.

3.4 The effect of societal support on transition paths

From the previous sections it already became clear that the societal support function was of influence on the course of the transition. The support function itself however was taken as independent of the specific functioning φ. The societal interpretation

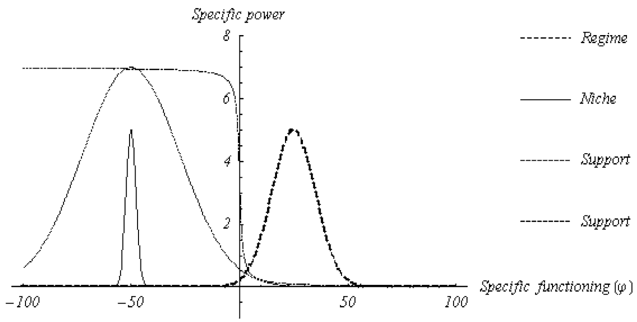

Fig. 4 Initial conditions with specific support functions

of this would be that a certain niche receives support, regardless of its functioning or future direction thereof. This sounds more ridiculous than it actually is, because subsidy schemes exist that do not discriminate on the sort of activities or research done by niche players but rather look at other criteria.

In the context of strategic niche management and transition management, however, it is relevant to know if it matters if support is given for regions of specific functioning. And if it is, in what way. For instance if the niche and the regime are distributed over the functional domain as in the previous example what are the consequences of a support function that only supports functioning for $\varphi < 0$? Or a support function that supports only the functioning that the niche already performs? Here these two forms are examined that is a support function of the form

$$S(\varphi) = 7\left(\frac{1}{2} + \frac{1}{\pi}\arctan(\varphi)\right), \tag{36}$$

and one of the form

$$S(\varphi) = 7e^{-\frac{(\varphi-\varphi_0)^2}{100}}. \tag{37}$$

Especially the second form would correspond to what (van den Bosch and Taanman 2006) call a strategy of ‘deepening’. This implies learning more and refining what the niche already does. See also Fig. 4. If the effect of these support functions is compared with the effect of the constant support with the same maximum value, 7, some counterintuitive things are to be noted. As in these cases support is given to a specific and therefore smaller region of functioning and with at most the same value, one would expect the niche to face more difficulties. This appears not the case. If the broadening parameter is varied from large (flexible) to small (rigid) the point at which the niche wins indeed is much further away. Stronger even, the point at which the niche took over in the case of constant support is close to the point where the niche merely survives in the specific support cases. This is made up for by the following, the niche can safely adopt a very rigid strategy. The niche then does not waste functioning in competition with the regime, since support is only given where the niche already functions and the regime is (almost) absent. For the transition as a whole this implies that when a niche adopts such a strategy and holds on the change in functioning is much more profound. As one can see in Fig. 5.

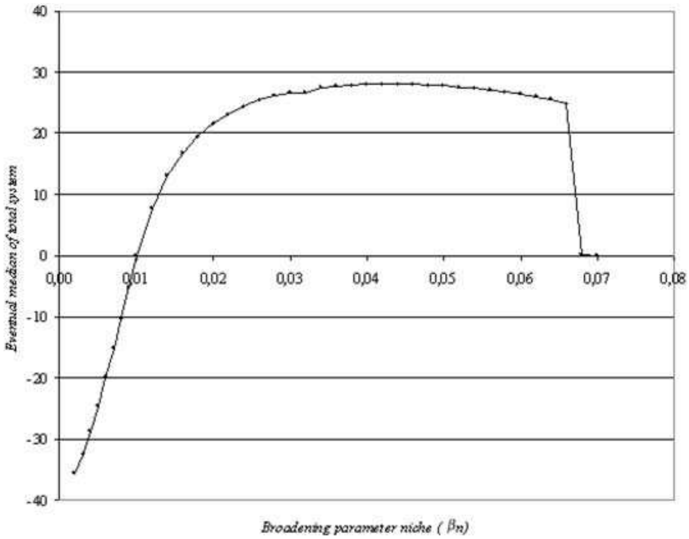

Fig. 5 Transitions with a locally supported niche

This works for the both forms of the support function with the best results with the form of (37).

4 Conclusions and discussion

Two types of conclusions can be drawn from this article. The first type concerns the contribution that this spatial approach to societal transitions is to this young field. In other words, how useful is the use of this approach in producing models based on current theoretical insights. The second is the converse, that is, what new insights for transition science have been gained by using this approach.

To start with the first, by basing everything on the parameterisation of the concept of functioning the framework allowed for a mathematical description of some central transition theory concepts such as, regime and niches. Other transition concepts such as 'broadening', 'deepening' and 'scaling up' appeared naturally as parameters in the equations and even power entered in a straightforward way. The landscape that is commonly used to conceptualise (semi-)exogenous influences in transition studies (see e.g. Rip and Kemp 1998) was mysteriously absent however, instead the environment of the system was modelled in terms of societal support only. The addition of a landscape appears straightforward though.

On the methodological side of things it connects with the spatial approaches that have been proven insightful in political science and simultaneously with a rich field in applied mathematics. One of the obvious disadvantages of this framework is also methodological in nature in that it is difficult to make sensible parameterisations of functioning in real transition cases. Although an example has been given in the text using personal (auto)mobility system, this remains problematic. This is also the reason that the second type of conclusions, the new insights, is best drawn with caution and modesty. The assumptions are crude and interpretation still shaky and at least some historical or real-life cases would have to be treated with this framework to find more realistic forms for the dynamical equations as well as getting grip on the various parameters.

However, the way for instance the influence of the broadening parameter and the form of the support function influence the course of the transitions in the model runs, could potentially be very useful and insightful in the context of strategic niche management and transition management, since both approaches to steering deal with niches that need to scale up and possibly change or take over the regime. Simple hypotheses like ‘it is good (or not) to support specific functioning for a transition to sustainability’ can be put to the test under various modelling assumptions. In the abstract some of these simple hypotheses have been evaluated in this article.

A pde approach like the one presented in this article has elegance in that all assumptions and hypotheses are compounded in a set of evolution equations and the initial and boundary conditions. It is in principle possible to glance at the equations and to know what the model is meant to model. But the drawback for this clarity is that the mathematical machinery is not that simple and to translate a hypothesis about how niches and regimes interact into a term in an equation demands some prior insight, or rather mathematical intuition.

Difficulties arise when one wants to increase the dimensionality. Throughout this article one dimension was used. For a realistic characterisation of the functioning of a societal system, however, probably several dimensions are necessary. This is no formal problem for the framework, but the tractability of the equations becomes difficult, first and foremost for analytical treatment, but simulation and numerical approaches as well suffer in more dimensions. Then again, in the political pendants in two dimensions already a lot of insight was gained. Expanding the model to feature more than just a regime and one niche is however rather easy.

Another difficulty is that although in this framework a niche can scale up to a point where it has gained more power than the regime it would still ‘behave’ as a niche. Simply because it is stuck in its equation so to say. This could in future approached be solved by simulations in several parts, or a form of the equations where this nicheness and regime-ness are parameters. The latter could be done by letting the diffusion coefficient, or in the terminology of this article the broadening parameter, depend on the power of the constellation.

In using this method some new questions arose as well, or in any case took on a new guise. An example of this is the relative importance of a niche becoming the new most powerful constellation in the system. In some cases the functioning of the entire system shifted significantly while the niche perished in the process. It is also, and maybe even in the first place, this kind of questions that justify the use of methods like these. That is, for the rigorous exploration of theoretical assumptions and to serve as an conceptual laboratory for the transition scientist.

Acknowledgements The author would like to thank Jos Timmermans and Michel Schilperoord for the valuable discussions that led to many insights in both theoretical and practical ways. Also the interactions with the Matisse WP9 group helped in shaping some of the concepts used in this article. Mark de Kok deserves not to remain unmentioned for his efforts of formulating this framework in Lagrangian form, although this was beyond the scope of this article. Much gratitude goes out to the ExTraScience community and especially those present at the Paris meeting of summer 2007. In addition the author would like to thank the referees that scrutinously reviewed this article and provided ample comments and suggestions, which to the authors opinion has greatly improved the article.

References

Avelino F (2007) Power in transition. KSI working paper. Publications at www.ksinetwork.nl

Baggio C, Howard M, van Saarloos W (2004) Dynamics and stability of vortex-antivortex fronts in type-II superconductors. Phys Rev E 70:026209

Bergman N, Haxeltine A, Whitmarsh L, Köhler J, Schilperoord M, Rotmans J (2008) Modelling socio-technical transition patterns and pathways. J Artif Soc Soc Simul 11(3):7. http://jasss.soc.surrey.ac.uk/11/3/7.html

Cross MC, Hohenberg PC (1993) Pattern formation outside of equilibrium. Rev Mod Phys 65(3):851, LP-1112

de Haan J (2007) Pillars of change: a theoretical framework for transition models. Paper presented at ESEE 2007 conference: integrating natural and social sciences for sustainability, Leipzig, June 6 2007

Downs A (1957) An economic theory of democracy. Harper and Brothers, New York

Farlow SJ (1982) Partial differential equations for scientists and engineers, 2nd edn. Dover Publications, New York

Geels FW (2002) Understanding the dynamics of technological transitions: a co-evolutionary and socio-technical analysis. Ph.D. thesis, Twente University

Giddens A (1984) The constitution of society. University of California Press, Berkeley and Los Angeles

Haugaard M (ed) (2002) Power: a reader. Manchester University Press, Manchester, New York

Kemp R, Schot J, Hoogma R (1998) Regime shifts to sustainability through processes of niche formation: the approach of strategic niche management. Technol Anal Strateg Manag 10(2):175–195

Kollman K, Miller JH, Page SE (1992) Adaptive parties in spatial elections. Am Polit Sci Rev 86(4)

Landau LD, Lifshitz EM (1951) The classical theory of fields, course of theoretical physics, vol 2, fourth revised English edition (1975). Elsevier, Amsterdam

Laver M (2005) Policy and the dynamics of political competition. Am Polit Sci Rev 99(2)

Levinthal DA (1998) The slow pace of rapid technological change: gradualism and punctuation in technological change. Ind Corp Change 7(2)

Luhmann N (1984) Social systems (trans. Bednarz J Jr with Baecker D). Stanford University Press, Stanford

Mossberger K, Stoker G (2001) The evolution of urban regime theory: the challenge of conceptualization. Urban Aff Rev 36(6):810–834

Nelson RR, Winter SG (1977) In search of useful theory of innovation. Res Policy 6:36–76

Rip A, Kemp R (1998) Technological change. In: Rayner S, Malone EL (eds) Human choice and climate change, vol 2. Battelle Press, Columbus, pp 327–399

Rogers EM (2003) Diffusion of innovations, 5th edn. Free Press, New York

Rotmans J (2005) Societal innovation: between dream and reality lies complexity. Inaugural addresses research in management series. ERIM, Erasmus Research Institute of Management, Rotterdam

Schot J, Geels FW (2007) Niches in evolutionary theories of technical change: A critical survey of the literature. J Evol Econ 17(5):605–622

van den Bosch S, Taanman M (2006) How innovation impacts society: patterns and mechanisms through which innovation projects contribute to transitions. Paper presented at Innovation pressure conference, Tampere, Finland, 15–17 March 2006

van Saarloos W (2003) Front propagation into unstable states. Phys Rep 386:29–222

Wilson CA (2000) Policy regimes and policy change. J Public Policy 20(3):247–274

Hans de Haan obtained his master degree in theoretical physics from the University of Leiden. He is currently employed as PhD researcher with the Dutch Research Institute for Transitions of Erasmus University Rotterdam. His primary research interests are the development of theory for, and computational and mathematical approaches to, societal transitions.